Representation theory of Lie groups

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Here is a quick plan:

1) Some basic reminder on structure theory of reductive Lie groups (almost no proofs, mainly examples, highlighting what is different from reductive groups over any field). Max 2 talks.

2) Review of the representation theory of compact Lie groups and semisimple Lie algebras. These are the two main examples people used historically to then study representation theory of more complex objects.

3) Introduction of (g, K)-modules and all related terminology, giving always the example and most of the proofs in the case of SL_2. Classification of admissible irreducible (g, K)-modules, with all the buzzwords like principal series, discrete series etc etc. This Is the most important part of the seminar.

4) Further results we can decide among include, for example, the classification of discrete series (think fundamental blocks of all irreps), Harish Chandra Subquotient Theorem/Casselman Submodule Theorem, minimal K-types of irreps.

I promise no Langlands classification.

Some references are:

a) The notes by I.A. MacDonald doing the structure theory with plenty of examples for each definition/proof.

b) The book by Knapp which often simplifies the proofs by working only with SL_2.

c) The notes by Gaitsgory which isolate and minimize the Analysis.

d) The Sea of surveys by Vogan, Borel etc.

Unfortunately the first two are not online but the physical versions are in my office.